Dislodging students' misconceptions through the use of worked examples and self-explanation

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Introduction

Students do not enter the classroom as blank slates. In mathematics classes, research shows that students can enter the classroom holding misconceptions that have the strong potential to derail new learning (Brown, 1992; Chiu & Liu, 2004; Kendeou & van den Broek, 2005). This has enormous implications for classroom instruction. The presence of student misconceptions suggests teachers need to identify and target misconceptions and build up accurate conceptual knowledge all while still providing students with enough instruction and practice on the wealth of procedural skill that are required course components and likely targets of standardized testing. Researchers in the domains of cognitive development and cognitive science have identified an instructional technique which may be especially helpful in fitting all these needs: the use of worked examples with self-explanation prompts.

This research brief (1) introduces conceptual and procedural knowledge in the context of Algebra I coursework; (2) provides examples of misconceptions and clarifies the their negative outcomes; (3) explains the structure of and evidence base behind worked examples with self-explanation; and (4) describes work done in the context of the SERP-MSAN partnership to further advance the development of classroom ready tools Algebra teachers will be able to use. The purpose of this research brief is to lay the groundwork for readers to recognize the value of adopting teaching tools that incorporate worked examples with self-explanation.

Algebra I conceptual and procedural knowledge: partners in mind

There is a consistent recommendation that teachers focus on concepts in mathematics. The National Council of Teachers of Mathematics (2000) stresses the importance of conceptual understanding for learning in math and recommends alignment of facts and procedures with concepts to improve student learning. More recently, the National Mathematics Advisory Panel (2008) recommended helping students master both concepts and skills, and maintained that preparation for Algebra requires simultaneous development of conceptual understanding and computational fluency, as well as cultivation of students' skill at solving problems. As an indicator of the level of emphasis placed on conceptual understanding, the final report of the National Mathematics Advisory Panel (2008) uses the words "concept" or "conceptual" 87 times in 120 pages; in comparison, the word "procedure" or "procedural" is used fewer than 40 times.

Conceptual knowledge has been defined as "an integrated and functional grasp of mathematical ideas" (National Research Council, 2001, p. 118). Consistent with this and other research on learning in mathematics, conceptual knowledge can be viewed as recognizing and understanding the important principles or features of a domain as well as interrelations or connections between different pieces of knowledge in the domain (Carpenter, Franke, Jacobs, Fennema, & Empsom. 1998; Hiebert & Wearne, 1996; Rittle-Johnson & Star, 2007). In contrast, procedural knowledge is the ability to carry out a series of actions in order to solve a problem (Hiebert, 1986; Rittle-Johnson, Siegler, & Alibali, 2001). In short, procedural knowledge can be operationally defined as how to do something, and conceptual knowledge as an understanding of what features in the task mean; conceptual knowledge of those features collectively allows one to understand why the procedure is appropriate for that task.

Though conceptual and procedural knowledge are often discussed as distinct entities, they do not develop independently in mathematics and, in fact, lie on a continuum, which often makes them hard to distinguish (Star, 2005; Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001). This may be especially difficult in Algebra, where many new procedures are taught over the course of the year (e.g., solving equations, factoring, graphing lines, etc.). Given the nature of the content in Algebra courses, items designed to measure conceptual knowledge may have elements that resemble procedural tasks. However, the information extracted about students' knowledge is not about their ability to carry out procedures. For example, one could give students the graph of a line and ask them to find the slope (procedural knowledge), or one could give students the same graph and ask them how the slope would change if the x and y intercepts were reversed (conceptual knowledge). Similarly, one could provide a pair of fractions and ask students to add them (procedural knowledge), or one could ask students to compare the sizes of the fractions and think about what would happen if the numerators and denominators were reversed (conceptual). Furthermore, one could show students an algebraic equation and ask them to solve it (procedural knowledge), or one could ask whether that equation is equivalent (or has the same solution set) to another equation (conceptual knowledge). Thus, even with the same stimulus for a problem, one can acquire very different types of information about what students know by the way that one asks them to think about the problem.

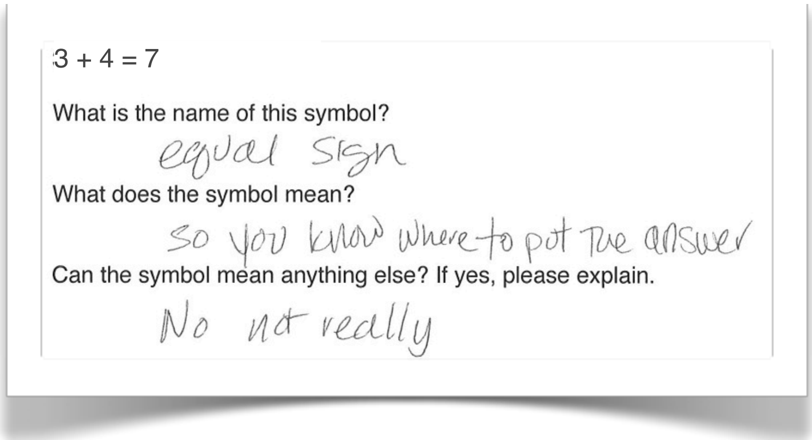
Misconceptions and their negative outcomes

For the past several decades, researchers in the fields of cognitive development and mathematics education have maintained that students beginning Algebra do not fully understand important concepts that teachers may expect them to have mastered from their elementary math and pre-algebra courses. Within the domain of equation solving alone, a number of concerning misconceptions have been identified, including that students believe that the equals sign is an indicator of operations to be performed (Baroody & Ginsburg, 1983; Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006), that negative signs represent only the subtraction operation and do not modify terms (Vlassis, 2004), that subtraction is commutative (Warren, 2003), and that variables cannot take on multiple values (Booth, 1984; Küchemann, 1978; Knuth et al., 2006). (See Figure 1 for examples of student misconceptions.) Unfortunately, for many students, these misconceptions persist even after traditional classroom instruction on the relevant topic (Vlassis, 2004; Booth, Koedinger, & Siegler, 2007).

How do these strange conceptions develop, and why are they so persistent? Misconceptions may have been ingrained in students due to particularities in the nature of their arithmetic instruction (Baroody & Ginsburg, 1983; Carpenter, Franke, & Levi, 2003; McNeil & Alibali, 2005). For example, the misconception that the equals sign indicates where the answer goes is likely due, at least in part, to the way math facts and early addition problems are presented by teachers and in textbooks. Such problems are often presented vertically, with one number on top of the other, and then a solid line between the addends and the answer. When students are given horizontally presented problems, they are typically in a format such as $4 + 5 = 9$, with numbers and operations appearing to the left side of the equals sign, and the answer (or a blank space for the answer) on the right side; students are rarely, if ever, exposed to other formats such as $9 = 4 + 5$ (Seo & Ginsburg, 2003) or $4 + 5 = 3 + 6$ (McNeil et al., 2006).

Figure 1: Student misconceptions about three key algebraic concepts: the equals sign, negative numbers, and variables and like terms.

Here, the student correctly names the equals sign, but then indicates that its only meaning is to show you where the answer goes. He/She doesn't understand the role of the equals sign in demonstrating / requiring balance on the two sides of an equation.


3 + 4 = 7

What is the name of this symbol?

equal sign

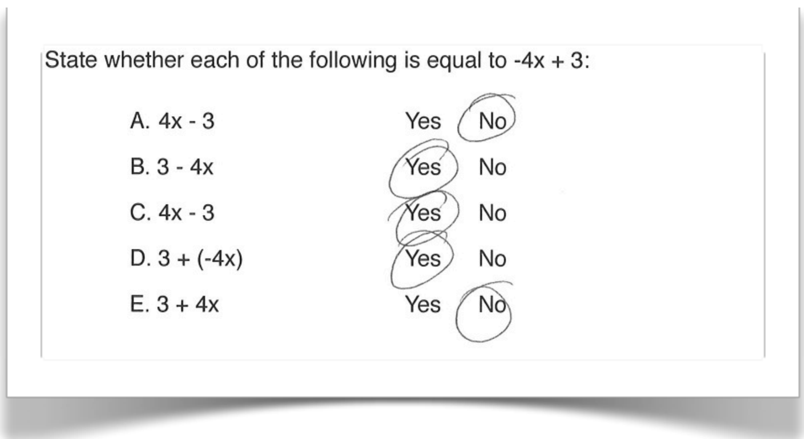
What does the symbol mean?

so you know where to put the answer

Can the symbol mean anything else? If yes, please explain.

No not really

This student correctly endorses items "b" and "d," and correctly rejects "a" and "e," but incorrectly endorses (c) 4x – 3 as equivalent to -4x + 3. The work suggests he/she misunderstands how negative signs modify numbers.


State whether each of the following is equal to -4x + 3:

A. 4x - 3 Yes No

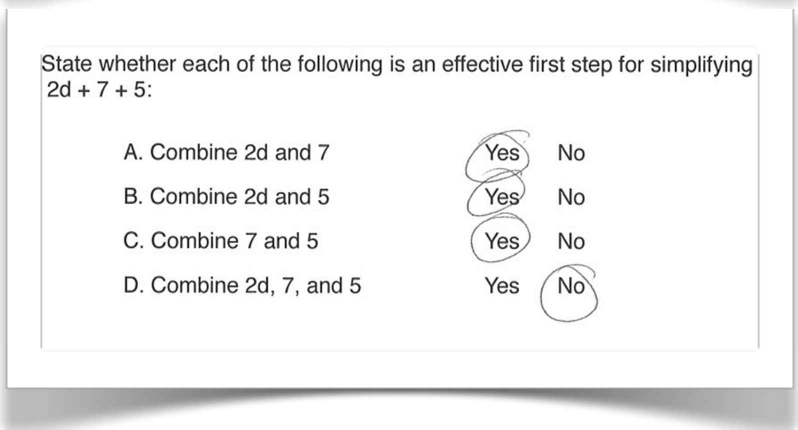
B. 3 - 4x Yes No

C. 4x - 3 Yes No

D. 3 + (-4x) Yes No

E. 3 + 4x Yes No

In responses to "a" and "b" this student demonstrates the misconception that unlike terms can be combined. Since in those items one term is a variable and one is a constant, combining them is not an acceptable first step.


State whether each of the following is an effective first step for simplifying 2d + 7 + 5:

A. Combine 2d and 7 Yes No

B. Combine 2d and 5 Yes No

C. Combine 7 and 5 Yes No

D. Combine 2d, 7, and 5 Yes No

McNeil (2008) found that even having students practice simple arithmetic problems in the typical format (4 + 5 = 9) as opposed to non-standard presentations (28 = 28) increased failure at mathematical equivalence problems (e.g., 3 + 5 + 6 = ___ + 6). Just imagine how much exposure to misleading problem formats students have gotten before they reach their Algebra 1 class, and how that might prompt them to approach algebraic equations!

As you might predict, these types of misconceptions are detrimental to students' performance on equation-solving tasks: students who hold misconceptions about critical features in algebraic equations solve fewer problems correctly (Booth & Koedinger, 2008). Even more interesting, these misconceptions are associated with the use of particular, related, but incorrect strategies when students attempt to solve problems. For example, students who do not think of negative signs as connected in any way to the subsequent numerical term often delete or move negatives within equations or subtract a term from both sides of the equation to eliminate the term even when the value in question is already negative; similarly, students who do not think of the equals sign as an indicator of balance between the terms on either side often delete or move the equals sign, or perform operations to only one side of the equation (Booth & Koedinger, 2008).

More crucially, such misconceptions also hinder students' learning of new material. Students who begin an equation-solving lesson with misconceptions learn less from a typical algebra lesson than students with more sound conceptual knowledge (Booth & Koedinger, 2008). Why might this be the case? One reason is highly related to abundant research in science education that demonstrates the importance of engaging and correcting students' preconceptions about scientific topics before presenting new information (Brown, 1992; Chiu & Liu, 2004). If these preconceptions are not engaged, teachers are just attempting to pile more information on top of a flawed foundation built on persistent misconceptions. In this case, students will not achieve full comprehension of the new material (Kendeou & van den Broek, 2005); rather, they may reject the new information that does not fit with their prior conception or try in vain to integrate the new information into their flawed or immature conceptions, resulting in a confused understanding of the content (Linn & Eylon, 2006). Further, recall that struggling students may not correctly encode the features of the equations they are presented by their teacher and their textbook (e.g., Booth & Davenport, in preparation). How can students be expected to learn what the teacher intends if they are not correctly viewing, let alone interpreting, the instructional materials? Eliminating student misconceptions should be a critical goal for successful mathematics instruction.

While this goal seems straightforward, with a limited amount of previous classroom time, how can teachers even hope to accomplish all of these goals? It would be nice if they were able to spend a day, or even a week of their Algebra course on helping their students gain a deep understanding of the equals sign, but doing so would prevent getting to the lessons on quadratics at the end of the year. Teachers need ways of improving conceptual understanding without sacrificing attention to procedural skills – and these ways need to easily incorporated into the many different algebra curricula used in classrooms across the country.

Worked Examples with Self-Explanation

Fortunately, some especially helpful instructional techniques have already been identified by researchers in the domains of cognitive development and cognitive science. One of these techniques is the use of worked examples with self-explanation prompts. Worked examples are just what they sound like—examples of problems worked out for students to consider, rather than for them to solve themselves (Sweller & Cooper, 1985). Replacing many of the problems in a practice session with examples of how to solve a problem leads to the same amount of procedural learning in less time (Zhu & Simon, 1987; Clark & Mayer, 2003), or increased learning and transfer of knowledge in the same amount of time (Paas, 1992).

When studying worked examples, students should be prompted to explain them using self-explanation prompts. Self-explanation facilitates students in integrating new information with what they already know, and forces the learner to make their new knowledge explicit (Chi, 2000; Roy & Chi, 2005). Typically, students are shown a correct example and asked to explain why the solution is correct. However, explaining a combination of correct and incorrect examples (i.e., explain why a common incorrect strategy is wrong) can be even more beneficial than explaining correct examples alone (Siegler, 2002; Siegler & Chen, 2008; Rittle-Johnson, 2006; Grosse & Renkl, 2007). Well-designed incorrect examples anticipate common misconceptions that students may hold that would make solving a particular type of problem difficult. For example, students may have a strategy that is perfectly good for some problems (e.g., combine two terms by adding the numbers involved; 4x + 3x is 7x), but misconceptions about the nature of variable vs. constant terms lead them to generalize this strategy to other problems where it is not appropriate (e.g., 4x + 3 is not 7x). When students study and explain incorrect examples, they directly confront these faulty concepts and are less likely to acquire or maintain incorrect ways of thinking about problems (Siegler, 2002; Ohlsson, 1996).

The worked example/self-explanation approach improves conceptual understanding without harming development of correct procedures. Many studies have established the benefits for procedural knowledge of worked examples (e.g., Sweller & Cooper, 1985; Zhu & Simon, 1987), and the benefits for conceptual understanding of self-explanation (e.g., Chi, 2000). Further, recent studies have shown that comparison and explanation of multiple correct examples (Rittle-Johnson & Star, 2009) or explanation of a combination of correct and incorrect examples (Booth, Paré-Blagoev, & Koedinger, 2010) can lead to both improved conceptual and procedural knowledge.

Making Worked Examples Work in the Classroom

The worked examples approach has been recommended for instructional use by the U.S. Department of Education (Pashler et al., 2007). However, this and other research-proven techniques often fail to find their way into everyday classroom practices or textbooks. This may be because education stakeholders do not believe that they will be useful in real-world classrooms, or perhaps because they see them as incompatible with the set-up of typical American classrooms. Further, necessary testing in classroom settings isn’t always feasible. However, greater collaboration between teachers, education researchers, and curriculum designers may be one way that true change can occur.

In 2006, a set of MSAN districts embarked on a partnership with the Strategic Education Research Partnership (SERP) Institute. One of the SERP-MSAN Field Site projects is the creation of a year’s worth of strategically designed Algebra I assignments that address student misconceptions and advance student learning. The *AlgebraByExample* materials created by this partnership interleave problems students must solve with worked examples that require self-explanation. Although results from myriad laboratory studies have been published demonstrating positive benefits of this and related approaches, only two previous publications included studies that were conducted in an actual classroom setting. The studies in those publications consisted of single classroom lessons. In contrast, the research undertaken by the SERP-MSAN partnership has taken place in more than 300 classrooms for durations of one-month to one year, and more than 6,000 students have participated. All studies included random assignment. In one condition students received *AlgebraByExample* assignments consisting 3-4 sets, which included a worked example and corresponding self-explanation prompt(s) and a similar problem for students to solve on their own. Students in the control condition received assignments that covered the same math and had the same number of items, however all problems were of the traditional type, i.e. no worked examples or prompts were provided. Additionally, all of the 150+ participating teachers had students in both groups to ensure teacher bias and teaching style was controlled for.

Results from these rigorous studies have consistently demonstrated that these materials support struggling students to make significant gains in their conceptual knowledge compared to those in control groups. Further, procedural gains are similar across groups although students using *AlgebraByExample* materials spend half of the time studying examples and the other half practicing doing a problem on their own "from scratch". In a year-long study conducted across 5 districts with 28 classrooms, struggling students scored an average of 10 percentage points higher in conceptual knowledge than peers in control classrooms taught by the same teacher.

Through this effort we have been able to examine how to implement this approach in real-world classrooms taking into account the heterogeneous constraints of multiple school districts. The SERP-MSAN partnership has made a new and significant contribution to integrate and advance what is known from research and practice in this realm. A concise summary of the types of studies conducted and findings are presented on the following page.

The final product is a set of materials that are deeply grounded in decades of laboratory studies and which have been through multiple intensive studies in eight different MSAN districts. They are supportive of Common Core Content and Practice Standards. Consistent with SERP's approach, the materials are also available digitally for free download. Freely downloaded materials can be printed by teachers, schools or districts. Other options are available if a school or district prefers to order and pay for printing through SERP. There are no content differences between the freely downloaded materials and those that may be ordered for delivery, the pay-for-printing option is offered as a convenience.

One might say that the previous laboratory studies have provided much of the conceptual knowledge necessary to justify the use of worked examples in classrooms. However, such studies could never provide the procedural knowledge of how to use worked examples in the classroom. It has taken the concerted efforts of everyone involved in the SERP-MSAN field-site work to create the combined conceptual and procedural knowledge necessary to develop a set of materials that is research based and classroom ready.

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This Research Brief includes significant excerpts from Booth, J.L (2011). Why can't students get the concept of math? *Perspectives on Language and Literacy,* Spring 2011, 31-35.

Study Type	Findings*	Design changes (implemented following year)
2008 - 2009		
Proof of Concept Studies on Prototype Assignments **Short pilot studies with 3-4 assignments completed in class** **Year long design focused study with up to 24 assignments completed either in class or at home.**	• AlgebraByExample leads to improved Conceptual scores and equivalent Procedural Scores • Non-Asian minority students benefit more from examples than other students • AlgebraByExample improves the learning trajectory of low-knowledge students • Completing more examples yields higher procedural gains; completing more problems does not. • Students who receive examples make fewer common errors when solving equations • In control classrooms, pre-test verbal skill predicts success. This is not the case in AlgebraByExample classrooms – likely because examples with self-explanation support verbal skills.	• Assignments shortened from 12 items to 8 • Content of each assignment better focused to fit well with a single lesson plan • Unit topics changed. • Number of assignments increased from 24 to 42
2010 - 2011		
Short pilot studies with 3-4 assignments completed in class	• AlgebraByExample leads to improved procedural scores for Low SES students • For some content units, AlgebraByExample leads to improved Conceptual scores • Students using AlgebraByExample show gains in motivation related outcomes	• 9 unit topics to address adjusted to fit wider range of curricula • Content adjusted within each unit to better fit a wider range of curricula • Items refined to better address critical misconceptions
2011 - 2012		
Double unit studies of 6-14 assignments completed in class	• With more assignments used, AlgebraByExample leads to improved Conceptual scores and equivalent Procedural scores • Non-Asian minority students benefit more from AlgebraByExample even when prior ability is controlled as measured by the pre-tests	• Items refined to better address critical misconceptions • Assignments compiled into a bound workbook

Study Type	Findings*	Design changes (implemented following year)
2012 - 2013		
Year long study using final *AlgebraByExample* assignments. Up to 42 assignments completed in class	• Students in the AlgebraByExample group who began the year with lower pretest scores made substantial gains in conceptual knowledge compared to those who were in the control group. • Procedural gains were consistent across students in both groups although control students had twice the practice solving problems. • Students rated the AlgebraByExample assignments as well as control assignments despite the demand for increase explanation. • Most teachers found that students using AlgebraByExample required less teacher support to complete assignment items than those who were given the control assignments	

*All studies were completed with random assignment and one group of students received AlgebraByExample assignments while the control group received similar math problems to solve but no worked examples or probing questions.

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